HW 1-13 page 1

2.3 page 130 59, 69, 75*, 77, 79, 81, 83, 91, 93, 97, 101

Read 2.4 Chain Rule ✓

$\boxed{2.3}$ (59) Find the slope at the point

$$y = \frac{1+\csc x}{1-\csc x}\frac{(\sin x)}{(\sin x)} = \frac{\sin x + 1}{\sin x - 1}$$

$$y' = \frac{(\sin x - 1)(\cos x) - (\sin x + 1)(\cos x)}{(\sin x - 1)^2}$$

$$= \frac{\cos x((\sin x - 1) - (\sin x + 1))}{(\sin x - 1)^2} = \frac{-2\cos x}{(\sin x - 1)^2}$$

when $x = \frac{\pi}{6}$ slope is $\frac{-2\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}-1\right)^2} = \frac{-\sqrt{3}}{\frac{1}{4}}$

$= -4\sqrt{3}$

(69) $f(x) = \frac{8}{x^2+4} = 8(x^2+4)^{-1}$

$f'(x) = -8(x^2+4)^{-2}(2x)\Big|_{x=2} = -8(8)^{-2}(4) = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x-2)$

(75) $f(x) = \frac{x^2}{x-1}$

$$f'(x) = \frac{(x-1)(2x) - x^2(1)}{(x-1)^2} = \frac{2x^2 - 2x - x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$$

when is $x^2 - 2x = 0$?

$x(x-2) = 0$ when $x = 0$ or 2

so $(0,0)$ and $(2,4)$

(77) find eq of the tangents of $f(x) = \frac{x+1}{x-1}$, $x \neq 1$
parallel to $2y + x = 6$ (slope of $-1/2$

$$f'(x) = \frac{(x-1)-(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2} = -\frac{1}{2}$$

$4 = x^2 - 2x + 1$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x = -1$ or 3 so $(-1,0), (3,2)$

$y = -\frac{1}{2}(x+1)$ & $y - 3 = -\frac{1}{2}(x-3)$

(79) Verify $f'(x) = g'(x)$ — what is the relationship?

$$f(x) = \frac{3x}{x+2}$$

$$f'(x) = \frac{(x+2)3 - 3x}{(x+2)^2} = \frac{3x+6-3x}{(x+2)^2} = \frac{6}{(x+2)^2}$$

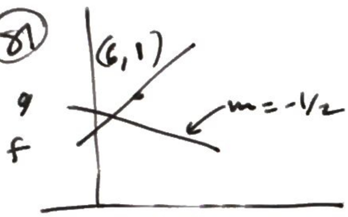
$$g(x) = \frac{5x+4}{x+2}$$

$$g'(x) = \frac{5(x+2)-(5x+4)}{(x+2)^2} = \frac{6}{(x+2)^2}$$

same slope
from graph it looks
like $g(x) = f(x) + 2$

check: $\frac{3x}{x+2} + 2\frac{(x+2)}{x+2} = \frac{3x+2x+4}{x+2} = g(x)$

(81)



(a) $p(x) = f(x)g(x)$

$$p'(x) = f(x)g'(x) + f'(x)g(x)$$

$$p'(1) = f(1)g'(1) + f'(1)g(1)$$

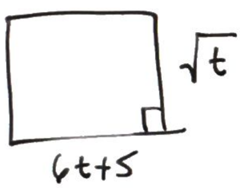
$$= (6)(-\tfrac{1}{2}) + (1)(4) = 1$$

(b) $q = \frac{f(x)}{g(x)}$

$$q'(x) = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$$

$$q'(4) = \frac{g(4)f'(4) - g'(4)f(4)}{[g(4)]^2} = \frac{(3)(-1)-(0)}{3^2} = \frac{-3}{9} = \frac{-1}{3}$$

(83)



$$A(t) = \sqrt{t}\,(6t+5) = 6t^{3/2} + 5t^{1/2}$$

$$A'(t) = 6\sqrt{t} + \frac{1}{2\sqrt{t}}(6t+5) \quad \text{(product rule)}$$

cm/sec

$$= 6\sqrt{t} + 3\sqrt{t} + \frac{5}{2\sqrt{t}}$$

$$= 9\sqrt{t} + \frac{5}{2\sqrt{t}}$$

HW 1-13 page 3

2.3 (p131) Find second derivatives

(91) $f(x) = x^2 + 7x - 4$

$f'(x) = 2x + 7$

$f''(x) = 2$

(93) $f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2}$

(97) $f(x) = x \sin x$

$f'(x) = x(\cos x) + (1)\sin x$

$f'(x) = \sin x + x \cos x$

$f''(x) = \cos x + [-x \sin x + (1)(\cos x)]$

$= \cos x - x \sin x + \cos x$

$= 2\cos x - x \sin x$

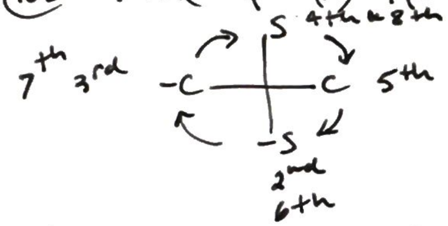
(101) Find $f^{(3)}(x)$ if $f'(x) = x^3 - x^{2/5}$

$f''(x) = 3x^2 - \frac{2}{5}x^{-3/5}$

$f^{(3)}(x) = f'''(x) = 6x + \frac{6}{25}x^{-8/5}$

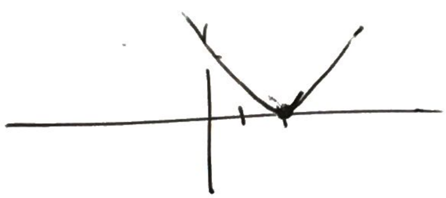
(103) Find $f^{(8)}(x)$ if $f''(x) = -\sin x$

$f^{(8)}(x) = \sin x$



(115) Sketch f where $f(2) = 0$, $f' < 0$ on $(-\infty, 2)$

$f' > 0$ on $(2, \infty)$



Could be $y = |x-2|$

but that is not diff'able

every where

maybe $(x-2)^2$